

$x^2 + px + q = 0$ を $(x+m)^2 = n$ の形に

2次方程式の $x^2 + px + q = 0$ は $(x+m)^2 = n$ の形に変形して
解くことができる、これを「平方完成」と言う。

① 次の方程式を解きましょう。

① $(x+3)^2 = 5$

$$\begin{aligned}x+3 &= \pm\sqrt{5} \\ x &= -3 \pm\sqrt{5}\end{aligned}$$

② $(x-5)^2 = 2$

$$\begin{aligned}x-5 &= \pm\sqrt{2} \\ x &= 5 \pm\sqrt{2}\end{aligned}$$

③ $(x-10)^2 - 24 = 0$

$$\begin{aligned}(x-10)^2 &= 24 \\ x-10 &= \pm\sqrt{24} \\ x &= 10 \pm 2\sqrt{6}\end{aligned}$$

④ $(x+2)^2 = 12$

$$\begin{aligned}x+2 &= \pm\sqrt{12} \\ x &= -2 \pm\sqrt{12} \\ x &= -2 \pm 2\sqrt{3}\end{aligned}$$

⑤ $(x+1)^2 - 36 = 0$

$$\begin{aligned}(x+1)^2 &= 36 \\ x+1 &= \pm\sqrt{36} \\ x &= -1 \pm 6 \\ x &= 5, -7\end{aligned}$$

⑥ $(x+8)^2 - 100 = 0$

$$\begin{aligned}(x+8)^2 &= 100 \\ x+8 &= \pm\sqrt{100} \\ x &= -8 \pm\sqrt{100} \\ x &= 2, -18\end{aligned}$$

② 次の方程式を $(x+m)^2 = n$ の形に変形して解きましょう。

① $x^2 + 6x - 2 = 0$

$$\begin{aligned}x^2 + 6x &= 2 \\ x^2 + 6x + 9 &= 2 + 9 \\ (x+3)^2 &= 11 \\ x+3 &= \pm\sqrt{11} \\ x &= -3 \pm\sqrt{11}\end{aligned}$$

② $x^2 + 4x - 2 = 0$

$$\begin{aligned}x^2 + 4x &= 2 \\ x^2 + 4x + 4 &= 2 + 4 \\ (x+2)^2 &= 6 \\ x+2 &= \pm\sqrt{6} \\ x &= -2 \pm\sqrt{6}\end{aligned}$$

③ $x^2 + 8x - 3 = 0$

$$\begin{aligned}x^2 + 8x &= 3 \\ x^2 + 8x + 16 &= 3 + 16 \\ (x+4)^2 &= 19 \\ x+4 &= \pm\sqrt{19} \\ x &= -4 \pm\sqrt{19}\end{aligned}$$

④ $x^2 + 3x - 1 = 0$

$$\begin{aligned}x^2 + 3x + \frac{9}{4} &= 1 + \frac{9}{4} \\ \left(x + \frac{3}{2}\right)^2 &= \frac{13}{4} \\ x &= \frac{-3 \pm\sqrt{13}}{2}\end{aligned}$$

⑤ $x^2 - 5x + 3 = 0$

$$\begin{aligned}x^2 - 5x + \frac{25}{4} &= -3 + \frac{25}{4} \\ \left(x - \frac{5}{2}\right)^2 &= \frac{13}{4} \\ x &= \frac{5 \pm\sqrt{13}}{2}\end{aligned}$$

⑥ $x^2 + 10x - 24 = 0$

$$\begin{aligned}x^2 + 10x &= 24 \\ x^2 + 10x + 25 &= 24 + 25 \\ (x+5)^2 &= 49 \\ x+5 &= \pm 7 \\ x &= 2, -12\end{aligned}$$